Bayesian Inference of Multiple Ising Models for Heterogeneous Data

Alejandra Avalos-Pacheco, PhD
@AleAviP
alejandra.avalos_pacheco@jku.at
JOHANNES KEPLER UNIVERSITÄT LINZ
Harvard-MIT Center for Regulatory Science

SISBAYES 2025

September 5^{th}, 2025

Andrea Lazzerini

Monia Lupparelli

Francesco C. Stingo

Journal of the Royal Statistical Society Series C: Applied Statistics, 2025, **00**, 1–32
https://doi.org/10.1093/jrsssc/qlaf028

ROYAL STATISTICAL SOCIETY
DATA | EVIDENCE | DECISIONS
Journal of the Statistics Society
Series C
Applied Statistics

Original Article

Bayesian inference of multiple Ising models for heterogeneous public opinion survey networks

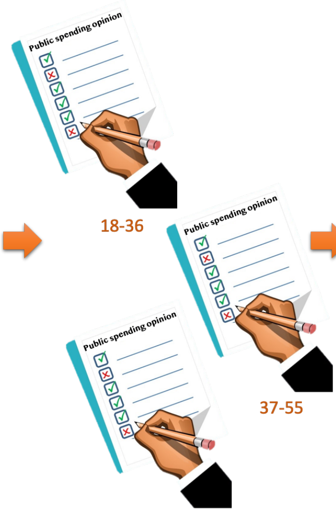
Alejandra Avalos-Pacheco[1,2], Andrea Lazzerini[3], Monia Lupparelli[3] and Francesco C. Stingo[3]

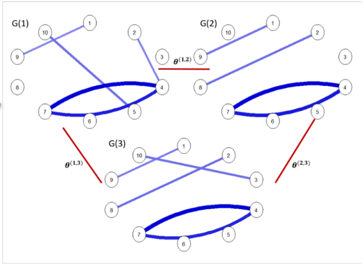
[1]Institute of Applied Statistics, Johannes Kepler University Linz, Altenberger Straße 69, Linz 4040, Austria
[2]Harvard-MIT Center for Regulatory Science, Harvard University, 200 Longwood Ave, Boston, MA 02115 USA
[3]Department of Statistics, Computer Science, Applications "G. Parenti", University of Florence, Viale Morgagni 59, Florence 50134, Italy

Address for correspondence: Alejandra Avalos-Pacheco, Institute of Applied Statistics, Johannes Kepler University Linz, Altenberger Straße 69, Linz 4040, Austria. Email: alejandra.avalos_pacheco@jku.at

Motivation

Name	Public spending opinion on
welf	welfare
high	highways and bridges
sec	social security
trans	mass transportation
park	parks and recreation
chi	assistance for childcare
sci	supporting scientific research
ern	developing alternative energy sources
for	foreign aid
mil	military, armaments and defense
bla	improving the conditions of African Americans
spa	space exploration program
env	improving and protecting environment
hea	improving and protecting nations health
cit	solving problems of big cities
crime	halting rising crime rate
drug	dealing with drug addiction
edu	improving nations education system





Multiple Ising Models

- Graphical models are effective tools to
  - model complex relationships in multivariate distributions of a set of variables
  - provide a graphical representation of their conditional independence structure

Multiple Ising Models

- Graphical models are effective tools to
  - model complex relationships in multivariate distributions of a set of variables
  - provide a graphical representation of their conditional independence structure
- Multiple graphical models for Gaussian random variables have been widely studied
  - Guo et al. (2011), Danaher et al. (2014), Peterson et al. (2015), Ha et al. (2021)
  - they can capture the heterogeneity of the data involved in more realistic settings

Multiple Ising Models

- Graphical models are effective tools to
  - model complex relationships in multivariate distributions of a set of variables
  - provide a graphical representation of their conditional independence structure
- Multiple graphical models for Gaussian random variables have been widely studied
  - Guo et al. (2011), Danaher et al. (2014), Peterson et al. (2015), Ha et al. (2021)
  - they can capture the heterogeneity of the data involved in more realistic settings
- There have been only few proposals for multinomial sampling models
  - Hojsgaard (2003), Corander (2003), Nyman et al. (2014, 2016).

Multiple Ising Models

- Graphical models are effective tools to
  - model complex relationships in multivariate distributions of a set of variables
  - provide a graphical representation of their conditional independence structure
- Multiple graphical models for Gaussian random variables have been widely studied
  - Guo et al. (2011), Danaher et al. (2014), Peterson et al. (2015), Ha et al. (2021)
  - they can capture the heterogeneity of the data involved in more realistic settings
- There have been only few proposals for multinomial sampling models
  - Hojsgaard (2003), Corander (2003), Nyman et al. (2014, 2016).
- Multiple Ising models:
  - More general, allow context-specific independences to vary not only with respect to adjacent vertices.
  - Model the heterogeneity induced in a set of binary variables by external factors.
  - Factors may influence the joint dependence relationships represented by a set of graphs across different groups.

Notation

Let

- $G(x) = (V, E(x))$: a graph with
- $V = \{1, 2, \dots, p\}$: a set of vertices
- $E(x)$: a set of edges which depends on $x \in \mathcal{X}$:
  - if $(r, j) \in E(x)$ there is an edge between r and j in $G(x)$
  - if $(r, j) \notin E(x)$ then the two nodes are disjointed in $G(x)$.
- $Y_V \mid X = x$: a random vector
- X: a random categorical variable or *external factor* not included in V.
  Variable X has q different levels $x \in \mathcal{X}$, or *profiles*.

Missing edges in G correspond to conditional independencies for the joint distribution of $Y_V \mid X = x$, i.e. $r, j \notin E(x) \implies Y_r \perp\!\!\!\perp Y_j \mid \{Y_{V\setminus(i,j)}, X = x\}$
Goal: Explore how $Y_V \mid X = x$ changes under different profiles

Bayesian model selection for Ising multiple graphs

Multiple undirected graphs for binary data.

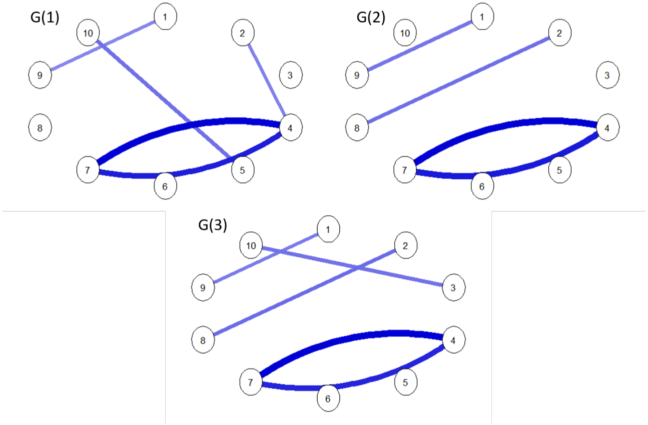


Figure: Example with $p = 10$ variables and $q = 3$ levels of X

Model

Y_V are binary random variables

$$Y_V(x) = Y_V \mid X = x \sim \text{Ising}(\lambda_x)$$

with canonical parameter $\lambda_x = [\lambda_{rj,x}]_{r,j \in V} \in \mathbb{R}^{p+(p \times (p-1))/2}$.

Model

Y_V are binary random variables

$$Y_V(x) = Y_V \mid X = x \sim \text{Ising}(\lambda_x)$$

with canonical parameter $\lambda_x = [\lambda_{rj,x}]_{r,j \in V} \in \mathbb{R}^{p+(p \times (p-1))/2}$.
Missing edges in $G(x)$ correspond to zero pairwise log-linear interactions, since $Y_r \perp\!\!\!\perp Y_j | \{Y_{V \setminus r,j}, X = x\}$ if and only if $\lambda_{rj,x} = 0$, for any $x \in \mathcal{X}$

Model

Y_V are binary random variables

$$Y_V(x) = Y_V \mid X = x \sim \text{Ising}(\lambda_x)$$

with canonical parameter $\lambda_x = [\lambda_{rj,x}]_{r,j \in V} \in \mathbb{R}^{p+(p\times(p-1))/2}$.
Missing edges in $G(x)$ correspond to zero pairwise log-linear interactions, since $Y_r \perp\!\!\!\perp Y_j | \{Y_{V\setminus r,j}, X = x\}$ if and only if $\lambda_{rj,x} = 0$, for any $x \in \mathcal{X}$
The likelihood is:

$$l(\lambda_x \mid Y_V(x)) = \prod_{i=1}^{n_x} \frac{1}{\Psi(\lambda_x)} \exp\left\{ \sum_{r=1}^{p} \lambda_{rr,x} y^i_{r,x} + \sum_{r=1}^{p} \sum_{j<i} \lambda_{rj,x} y^i_{r,x} y^i_{j,x} \right\}$$

with $y^i_{r,x}$ the i^{th} row of $Y_V(x)$.

Model

Y_V are binary random variables

$$Y_V(x) = Y_V \mid X = x \sim \text{Ising}(\lambda_x)$$

with canonical parameter $\lambda_x = [\lambda_{rj,x}]_{r,j \in V} \in \mathbb{R}^{p+(p\times(p-1))/2}$.
Missing edges in $G(x)$ correspond to zero pairwise log-linear interactions, since $Y_r \perp\!\!\!\perp Y_j | \{Y_{V\setminus r,j}, X = x\}$ if and only if $\lambda_{rj,x} = 0$, for any $x \in \mathcal{X}$
The likelihood is:

$$l(\lambda_x \mid Y_V(x)) = \prod_{i=1}^{n_x} \frac{1}{\Psi(\lambda_x)} \exp\left\{ \sum_{r=1}^{p} \lambda_{rr,x} y^i_{r,x} + \sum_{r=1}^{p} \sum_{j<i} \lambda_{rj,x} y^i_{r,x} y^i_{j,x} \right\}$$

with $y^i_{r,x}$ the i^{th} row of $Y_V(x)$.
Challenge: $\Psi(\lambda_x)$ is hard to compute.

Normalizing constant $\Psi(\lambda_x)$

Challenge: $\Psi(\lambda_x)$ is hard to compute.

$$\Psi(\lambda_x) = \sum_{y_x^i \in \{0,1\}^p} \exp\left\{ \sum_{r=1}^{p} \lambda_{rr,x} y_{r,x}^i + \sum_{r=1}^{p} \sum_{j<r} \lambda_{rj,x} y_{r,x}^i y_{j,x}^i \right\}$$

Solutions:

1. Conjugate priors for low-dimensional graphs: Fully Bayesian (FB)
2. Quasi-likelihood approach for high-dimensional graphs: Approximate Bayesian (AB) (Bhattacharyya and Atchade 2019)

Approximate Bayesian (AB)

Express the r^{th} node conditional likelihood for $\lambda_{r,x} \in \mathbb{R}^p$, $r \in V$ and for any $x \in \mathcal{X}$, as

$$p_r(Y_V(x)|\lambda_{r,x}) = \prod_{i=1}^{n_x} \frac{1}{\Psi^i_{r,x}(\lambda_{r,x})} \exp\left\{ \lambda_{rr,x} y^i_{r,x} + \sum_{j<r} \lambda_{rj,x} y^i_{r,x} y^i_{j,x} \right\},$$

Approximate Bayesian (AB)

Express the r^{th} node conditional likelihood for $\lambda_{r,x} \in \mathbb{R}^p$, $r \in V$ and for any $x \in \mathcal{X}$, as

$$p_r(Y_V(x)|\lambda_{r,x}) = \prod_{i=1}^{n_x} \frac{1}{\Psi^i_{r,x}(\lambda_{r,x})} \exp\left\{ \lambda_{rr,x} y^i_{r,x} + \sum_{j<r} \lambda_{rj,x} y^i_{r,x} y^i_{j,x} \right\},$$

with a normalization constant equal to

$$\Psi^i_{r,x}(\lambda_{r,x}) = 1 + \exp\left\{ \lambda_{rr,x} + \sum_{j<r} \lambda_{rj,x} y^i_{j,x} \right\}, \qquad x \in \mathcal{X}.$$

Approximate Bayesian (AB)

Express the r^{th} node conditional likelihood for $\lambda_{r,x} \in \mathbb{R}^p$, $r \in V$ and for any $x \in \mathcal{X}$, as

$$p_r(Y_V(x)|\lambda_{r,x}) = \prod_{i=1}^{n_x} \frac{1}{\Psi^i_{r,x}(\lambda_{r,x})} \exp\left\{\lambda_{rr,x} y^i_{r,x} + \sum_{j<r} \lambda_{rj,x} y^i_{r,x} y^i_{j,x}\right\},$$

with a normalization constant equal to

$$\Psi^i_{r,x}(\lambda_{r,x}) = 1 + \exp\left\{\lambda_{rr,x} + \sum_{j<r} \lambda_{rj,x} y^i_{j,x}\right\}, \quad x \in \mathcal{X}.$$

Approximate the likelihood with a quasi-likelihood:

$$p_q(Y_V(x) \mid \lambda_x) = \prod_{r=1}^{p} p_r(Y_V(x) \mid \lambda_{r,x}), \quad x \in \mathcal{X},$$

Approximate Bayesian (AB)

Express the r^{th} node conditional likelihood for $\lambda_{r,x} \in \mathbb{R}^p$, $r \in V$ and for any $x \in \mathcal{X}$, as

$$p_r(Y_V(x)|\lambda_{r,x}) = \prod_{i=1}^{n_x} \frac{1}{\Psi^i_{r,x}(\lambda_{r,x})} \exp\left\{\lambda_{rr,x} y^i_{r,x} + \sum_{j<r} \lambda_{rj,x} y^i_{r,x} y^i_{j,x}\right\},$$

with a normalization constant equal to

$$\Psi^i_{r,x}(\lambda_{r,x}) = 1 + \exp\left\{\lambda_{rr,x} + \sum_{j<r} \lambda_{rj,x} y^i_{j,x}\right\}, \quad x \in \mathcal{X}.$$

Approximate the likelihood with a quasi-likelihood:

$$p_q(Y_V(x) \mid \lambda_x) = \prod_{r=1}^{p} p_r(Y_V(x) \mid \lambda_{r,x}), \quad x \in \mathcal{X},$$

The inference on $\lambda_x \in \mathbb{R}^{\mathbf{p+(p\times(p-1))/2}}$ simplifies into p separable sub-problems on $\mathbb{R}^{\mathbf{p}}$.

$\lambda_{rj,x}$ Priors

Challenge: Encourage sparse graphs, for interpretability

$\lambda_{rj,x}$ Priors

Challenge: Encourage sparse graphs, for interpretability

Solution: Add sparsity-induced priors

$\lambda_{rj,x}$ Priors

Challenge: Encourage sparse graphs, for interpretability

Solution: Add sparsity-induced priors

1. For $p \leq 10$: Diaconis & Ylvisaker (1973) prior:

$$p(\lambda_x \mid \delta_x) = C(s_x, \alpha_x)^{-1} \exp\left\{\sum_r \lambda_{rr,x} s_{rr,x} + \sum_{r<j} \delta_{rj,x}\lambda_{rj,x} s_{rj,x} - g_x \log\left[\sum_{\{\mathcal{I}_x \setminus y_{\emptyset,x}\}} \exp\left(\sum_r \lambda_{rr,x} + \sum_{r<j} \delta_{rj,x}\lambda_{rj,x}\right)\right]\right\},$$

where $C(s_x, \alpha_x)$ is an unknown normalization constant that depends on the hyperparameters $g_x \in \mathbb{R}$ and $s_x = [s_{rj,x}]_{r,j \in V}$, $s_x \in \mathbb{R}^{p+(p\times(p-1))/2}$.

$\lambda_{rj,x}$ Priors

Challenge: Encourage sparse graphs, for interpretability
Solution: Add sparsity-induced priors

1. For $p \leq 10$: Diaconis & Ylvisaker (1973) prior:

$$p(\lambda_x \mid \delta_x) = C(s_x, \alpha_x)^{-1} \exp\left\{\sum_r \lambda_{rr,x} s_{rr,x} + \sum_{r<j} \delta_{rj,x}\lambda_{rj,x}s_{rj,x} - g_x \log\left[\sum_{\{\mathcal{I}_x \setminus y_{\emptyset,x}\}} \exp\left(\sum_r \lambda_{rr,x} + \sum_{r<j} \delta_{rj,x}\lambda_{rj,x}\right)\right]\right\},$$

   where $C(s_x, \alpha_x)$ is an unknown normalization constant that depends on the hyperparameters $g_x \in \mathbb{R}$ and $s_x = [s_{rj,x}]_{r,j \in V}$, $s_x \in \mathbb{R}^{p+(p\times(p-1))/2}$.

2. For $p > 10$: George & McCulloch (1993) Normal spike-and-Slab prior:

$$p(\lambda_{rj,x} \mid \delta_{rj,x}, \gamma_0, \gamma_1) = \delta_{rj,x} N(\lambda_{rj,x}; 0, \rho) + (1 - \delta_{rj,x}) N(\lambda_{rj,x}; 0, \gamma),$$

   with $\rho >> \gamma > 0$
   $\boldsymbol{\delta_{rj,x}} \in \{0, 1\}$ an indicator parameter for the inclusion of $\lambda_{rj,x}$.

$\delta_{rj,x}$ Prior

Challenge: Consider $\delta_{rj,x}$ such that $\lambda_{rj,x}$ considers the **similarities** among **all** graphs $G(x)$

$\delta_{rj,x}$ Prior

Challenge: Consider $\delta_{rj,x}$ such that $\lambda_{rj,x}$ considers the **similarities** among **all** graphs $G(x)$
Solution: Consider a Markov Random Field (MRF) prior (Peterson et al., 2015) on the graph structures

$$p(\delta_{rj,x} \mid \delta_{rj,-x}, \nu_{rj}, \theta_x) = \frac{\exp[\delta_{rj,x}(\nu_{rj} + \mathbf{1}^\top \theta_x \delta_{rj,x})]}{1 + \exp[\delta_{rj,x}(\nu_{rj} + \mathbf{1}^\top \theta_x \delta_{rj,-x})]}, \quad x \in \mathcal{X},$$

$\delta_{rj,x}$ Prior

Challenge: Consider $\delta_{rj,x}$ such that $\lambda_{rj,x}$ considers the **similarities** among **all** graphs $G(x)$
Solution: Consider a Markov Random Field (MRF) prior (Peterson et al., 2015) on the graph structures

$$p(\delta_{rj,x} \mid \delta_{rj,-x}, \nu_{rj}, \theta_x) = \frac{\exp[\delta_{rj,x}(\nu_{rj} + \mathbf{1}^\top \theta_x \delta_{rj,x})]}{1 + \exp[\delta_{rj,x}(\nu_{rj} + \mathbf{1}^\top \theta_x \delta_{rj,-x})]}, \quad x \in \mathcal{X},$$

$\nu_{rj} \in \mathbb{R}$ is a sparsity parameter specific for the edge (r, j) , $\nu_{rj} \sim \text{Beta}(a, b)$

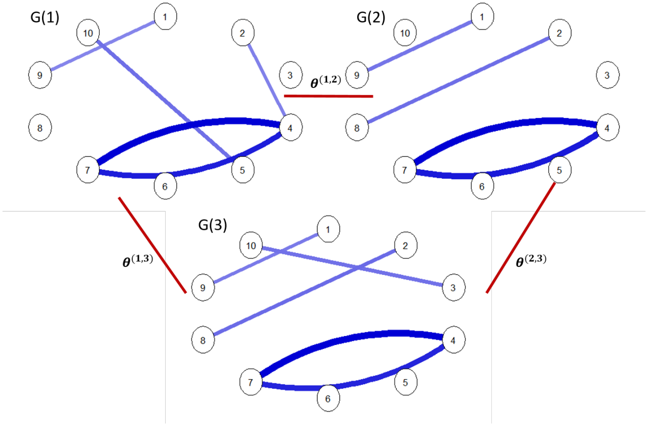
$\delta_{rj,x}$ Prior

Challenge: Consider $\delta_{rj,x}$ such that $\lambda_{rj,x}$ considers the **similarities** among **all** graphs $G(x)$
Solution: Consider a Markov Random Field (MRF) prior (Peterson et al., 2015) on the graph structures

$$p(\delta_{rj,x} \mid \delta_{rj,-x}, \nu_{rj}, \theta_x) = \frac{\exp[\delta_{rj,x}(\nu_{rj} + \mathbf{1}^\top \theta_x \delta_{rj,x})]}{1 + \exp[\delta_{rj,x}(\nu_{rj} + \mathbf{1}^\top \theta_x \delta_{rj,-x})]}, \quad x \in \mathcal{X},$$

$\nu_{rj} \in \mathbb{R}$ is a sparsity parameter specific for the edge (r, j) , $\nu_{rj} \sim \text{Beta}(a, b)$
$\theta_x = [\theta_{xh}]_{h \in \{\mathcal{X} \setminus x\}}$, where $\theta_{xh} \in \mathbb{R}$ represents the relatedness between $G(x)$ and $G(h)$
Note: that this MRF prior is an Ising model $\rightarrow$ greatly simplifies the computations

Network similarity



θ_{xh} Priors

Challenge: Measure pairwise similarity of the $G(x), G(h)$ with a flexible prior that:

- shares information across groups if they are truly similar,
- does not fakely enforce similarities when different.

θ_{xh} Priors

Challenge: Measure pairwise similarity of the $G(x), G(h)$ with a flexible prior that:

- shares information across groups if they are truly similar,
- does not fakely enforce similarities when different.

Solution: Model θ_{xh} with a George & McCulloch (1993) Spike-and-Slab prior, with a point mass spike:

$$p(\theta_{xh}) \mid \epsilon_{xh} = (1 - \epsilon_{xh})\delta_0 + \epsilon_{xh}\text{Gamma}(\theta_{xh} \mid \alpha, \beta),$$

with $\epsilon_{xh} \sim \text{Bernoulli}(\epsilon_{xh} \mid \omega)$.
Note: This is a Spike-and-non-local-Slab prior Johnson & Rossell (2010, 2012), Avalos-Pacheco et.al. (2022)

Posterior inference

while $t < T$ **do**

FB:

Update the graph $G(x)$ for each profile $x \in \mathcal{X}$
via stochastic search of the graph space, using a Laplace approximation
of the marginal likelihood

AB:

Update the graph $G(x)$ and the canonical parameter λ_x
sampling them from the conditional quasi-posterior distribution

FB & AB:

Update the graph similarity θ_{xh} and the latent indicators ϵ_{xh} for $1 \leq x < h \leq q$
via Metropolis-Hastings to sample them from their conditional posterior distribution

Perform: a between-model and a within-model move (to improve mixing)

Update the edge-specific parameter ν_{rj} for $1 \leq r < j \leq p$.
via Metropolis-Hastings to sample them from their conditional posterior distribution

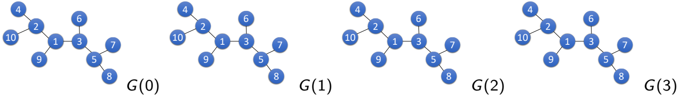
set $t = t + 1$

end while

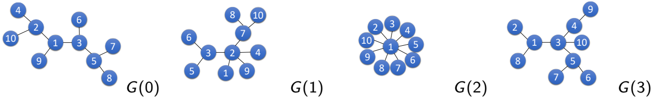
R-code available at: `https://github.com/AleAviP/BMIM`

Simulation results

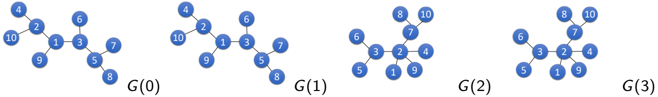
Scenario (A)



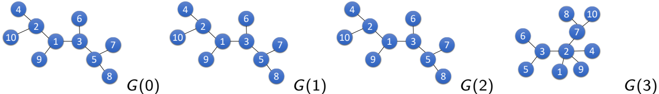
Scenario (B)



Scenario (C)



Scenario (D)



Simulation results: low dimensional data ($p = 10$)

Matthews correlation coefficient				
Model	Scenario (A)	Scenario (B)	Scenario (C)	Scenario (D)
SL	0.773(0.048)	0.811(0.058)	0.739(0.062)	0.762(0.067)
DSSL	**0.983**(0.020)	0.589(0.060)	0.518(0.087)	0.714(0.040)
ABS	0.734(0.070)	0.761(0.058)	0.674(0.078)	0.712(0.070)
AB	0.814(0.036)	0.808(0.060)	0.744(0.050)	0.772(0.057)
FBS	0.796(0.038)	**0.822**(0.070)	0.749(0.054)	0.785(0.049)
FB	0.858(0.042)	0.804(0.064)	**0.764**(0.074)	**0.812**(0.041)

F1 score				
Model	Scenario (A)	Scenario (B)	Scenario (C)	Scenario (D)
SL	0.812(0.040)	0.838(0.052)	0.770(0.055)	0.794(0.062)
DSSL	**0.986**(0.017)	0.665(0.057)	0.614(0.074)	0.774(0.033)
ABS	0.753(0.068)	0.778(0.051)	0.685(0.074)	0.728(0.068)
AB	0.839(0.035)	0.828(0.048)	0.769(0.048)	0.797(0.056)
FBS	0.825(0.035)	**0.844**(0.066)	0.781(0.047)	0.812(0.044)
FB	0.880(0.036)	0.830(0.055)	**0.792**(0.065)	**0.833**(0.043)

- Seplogit (SL): Meinshausen and Bühlmann (2006)
- DataShared-SepLogit (DSSL): Ollier and Viallon. (2017)

Simulation results: high dimensional data ($p = 50$)

Matthews correlation coefficient				
Model	Scenario (A)	Scenario (B)	Scenario (C)	Scenario (D)
SL	0.916(0.010)	0.911(0.011)	0.915(0.012)	0.916(0.008)
DSSL	**0.988**(0.007)	0.798(0.031)	0.740(0.017)	0.838(0.018)
ABS	0.920(0.010)	0.914(0.012)	0.910(0.010)	0.915(0.010)
AB	0.947(0.008)	**0.924**(0.011)	**0.931**(0.010)	**0.935**(0.009)

F1 score				
Model	Scenario (A)	Scenario (B)	Scenario (C)	Scenario (D)
SL	0.919(0.010)	0.914(0.010)	0.917(0.012)	0.919(0.007)
DSSL	**0.988**(0.007)	0.798(0.032)	0.731(0.021)	0.838(0.018)
ABS	0.922(0.010)	0.915(0.012)	0.911(0.010)	0.917(0.010)
AB	0.949(0.008)	**0.927**(0.011)	**0.933**(0.010)	**0.937**(0.009)

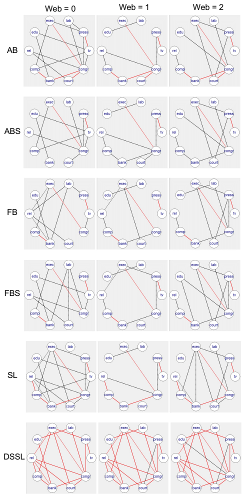
GSS datasets (https://gssdataexplorer.norc.org/)

1. **Confidence in government institutions**
   - study whether confidence in government agencies can be influenced by the user's time-spent on the web.
   - 10 binary variables that reflect the opinion of distrust (0) or at least some trust (1)
   - 450 individuals divided according weekly surfing time:
     - $Web = 0$ for at most 5 hours
     - $Web = 1$ for more than 5 hours and at most 15 hours
     - $Web = 2$ for more than 15 hours

GSS datasets (https://gssdataexplorer.norc.org/)

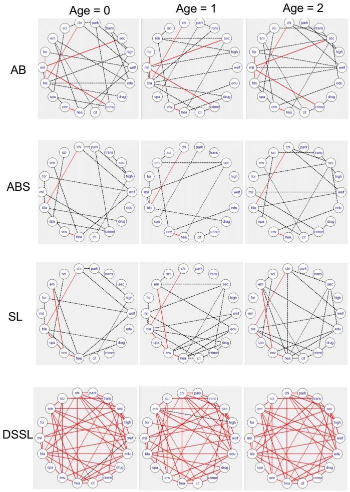
1. **Confidence in government institutions**
   - study whether confidence in government agencies can be influenced by the user's time-spent on the web.
   - 10 binary variables that reflect the opinion of distrust (0) or at least some trust (1)
   - 450 individuals divided according weekly surfing time:
     - $Web = 0$ for at most 5 hours
     - $Web = 1$ for more than 5 hours and at most 15 hours
     - $Web = 2$ for more than 15 hours
2. **Public spending opinion**
   - study if the opinions on public spending in US for some crucial social issues can be considerably divergent at different ages
   - 18 binary variables that reflect the opinion of underfunded (1) or not underfunded (0)
   - 768 individuals categorized by age:
     - $Age = 0$ age 36 or younger
     - $Age = 1$ older than 36 years and of age 55 or younger
     - $Age = 2$ for the ones older than 55 years.

Confidence in government institutions



expected FDR: ABS = 0.22, AB = 0.20, FBS = 0.14, FB = 0.12

Public spending opinion



expected FDR: ABS = 0.22, AB = 0.18

Conclusions

- Efficiently link and borrow strength across related sub-populations
- We have introduced two Bayesian multiple Ising graphical inference methods:
  1. The FB method
     - based on conjugate priors
  2. The AB method
     - based on a quasi-likelihood approach $\rightarrow$ computational non-expensive
- We use of a MRF prior on the binary indicator of edge inclusion
  - encourage the selection of the same edges in related graphs
  - can learn which sup-populations are similar, and which ones are not.
- We show the usefulness of our approaches in two public opinion studies in the US
- **R**-code available at: `https://github.com/AleAviP/BMIM`
- Alejandra Avalos-Pacheco, Andrea Lazzerini, Monia Lupparelli, Francesco C Stingo, *Bayesian inference of multiple Ising models for heterogeneous public opinion survey networks*, **Journal of the Royal Statistical Society Series C: Applied Statistics**, 2025; qlaf028, https://doi.org/10.1093/jrsssc/qlaf028

Thank you! Grazie!

@AleAviP
alejandra.avalos_pacheco@jku.at